

Class - 12TH
MATHEMATICS

MOCK EXAMS

Serial Number: 03

Time: 3 Hours

Maximum Marks: 80

Exam Date: _____



CLASS 12TH
MATHEMATICS

EXAM ID 0643

PAPER - 3

DURATION 3 HOURS

MM. 80

General Instructions:

Read the following instructions carefully and follow them:

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study-based questions carrying 4 marks each with sub-parts of the values of 1, 1 and 2 marks each respectively.
8. All Questions are compulsory. However, an internal choice in 2 Questions of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
9. Use of calculators is not allowed.

SECTION – A (1 MARKS EACH)

1. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then $(A + 2B)'$ is
a. $\begin{bmatrix} 1 & -4 \\ 6 & 5 \end{bmatrix}$ b. $\begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$ c. $\begin{bmatrix} -4 & 6 \\ 1 & 5 \end{bmatrix}$ d. $\begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$
2. Matrices A and B are inverses of each other only when
a. $AB = BA = O$ b. $AB = O, BA = I$ c. $AB = BA$ d. $AB = BA = I$
3. The value of the determinant $\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$ is:
a. 47 b. 49 c. -51 d. -79
4. If $y = 2^x$ then $\frac{dy}{dx} = ?$
a. $x(2^{x-1})$ b. $2^x(\log 2)$ c. $\frac{2^x}{(\log 2)}$ d. $3^x(\log 3)$
5. If the points A(-1, 3, 2), B(-4, 2, -2) and C(5, 5, λ) are collinear then the value of λ is
a. 10 b. 7 c. 5 d. 8
6. The degree of the differential equation $x^2 \frac{d^2y}{dx^2} = \left(x \frac{dy}{dx} - y\right)^3$ is
a. 2 b. 1 c. 3 d. 6
7. The corner points of the feasible region for a Linear Programming problem are P(0, 5), Q(1, 5), R(4, 2) and S(12, 0). The minimum value of the objective function $Z = 2x + 5y$ is at the point.
a. R b. Q c. S d. P
8. The principal value of the expression $\cos^{-1}[\cos(-680^\circ)]$ is
a. $\frac{\pi}{9}$ b. $\frac{2\pi}{9}$ c. $\frac{-2\pi}{9}$ d. $\frac{34\pi}{9}$
9. $\int_{-\pi}^{\pi} \sin^5 x dx = ?$
a. 2π b. $\frac{3\pi}{4}$ c. 0 d. $\frac{5\pi}{16}$

10. If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then the value of x, y is
 a. $x = 3, y = 1$ b. $x = 2, y = 3$ c. $x = 2, y = 4$ d. $x = 3, y = 3$
11. If the constraints in a linear programming problem are changed
 a. the problem is to be re-evaluated b. solution is not defined
 c. the objective function has to be modified d. the change in constraints is ignored
12. Let $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $|\vec{a} + \vec{b} + \vec{c}| = 1$ and $\vec{a}$ is perpendicular to $\vec{b}$. If $\vec{c}$ makes angle α and β with $\vec{a}$ and $\vec{b}$ respectively, then $\cos \alpha + \cos \beta =$
 a. $-\frac{3}{2}$ b. 1 c. -1 d. $\frac{3}{2}$
13. If $\mathbf{P} = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to
 a. 11 b. 4 c. 5 d. 13
14. The letters of the word ORIENTAL are arranged in all possible ways. The chance that the consonants and vowels occur alternately is
 a. $\frac{3}{35}$ b. $\frac{1}{35}$ c. $\frac{2}{35}$ d. $\frac{1}{70}$
15. If $x = a \cos \theta + b \sin \theta, y = a \sin \theta - b \cos \theta$, then which one of the following is true?
 a. $y^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 0$ b. $y^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - y = 0$
 c. $y^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = 0$ d. $y^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$
16. Which one of the following is the unit vector perpendicular to both $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$?
 a. $\frac{\hat{j} + \hat{k}}{\sqrt{2}}$ b. $\pm \frac{\hat{i} + \hat{j}}{\sqrt{2}}$ c. $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$ d. $\hat{k}$
17. Let $f(x) = x - |x|$ then $f(x)$ is
 a. Differentiable $\forall x \in \mathbb{R}$ b. continuous $\forall x \in \mathbb{R}$ and not differentiable at $x = 0$
 c. discontinuous at $x = 0$ d. neither continuous nor differentiable at $x = 0$
18. If a line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube then $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta$ is equal to
 a. $\frac{2}{3}$ b. $\frac{4}{3}$ c. $\frac{8}{3}$ d. $\frac{1}{3}$
19. **Assertion (A):** If x is real, then the minimum value of $x^2 - 8x + 17$ is 1.
Reason (R): If $f''(x) > 0$ at a critical point, then the value of the function at the critical point will be the minimum value of the function.
 a. Both A and R are true and R is the correct explanation of A.
 b. Both A and R are true but R is not the correct explanation of A.
 c. A is true but R is false.
 d. A is false but R is true.
20. **Assertion (A):** The modulus function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = |x|$ is neither one-one nor onto.
Reason (R): The signum function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$ is bijective.
 a. Both A and R are true and R is the correct explanation of A.
 b. Both A and R are true but R is not the correct explanation of A.
 c. A is true but R is false.
 d. A is false but R is true.

SECTION – B (2 MARKS EACH)

21. Evaluate: $\tan^{-1} \sqrt{3} - \sec^{-1}(-2)$.

OR

Write the interval for the principal value of function and draw its graph: $\tan^{-1} x$.

22. Find all the points of discontinuity of the greatest integer function defined by $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x .

23. Find the intervals in which $f(x) = (x + 2)e^{-x}$ is increasing or decreasing.

OR

If $x^{30}y^{20} = (x + y)^{50}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

24. Evaluate: $\int e^x (\cot x - \operatorname{cosec}^2 x) dx$.

25. If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$, find AA^T .

SECTION – C (3 MARKS EACH)

26. Evaluate: $\frac{1}{2\cos x + 4\sin x} dx$.

27. Differentiate the function $\cot^{-1} \left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right]$, $0 < x < \frac{\pi}{2}$ w.r.t. x .

28. Evaluate $\int \frac{dx}{x(x^5+3)}$.

OR

Evaluate the integral: $\int \frac{1}{\sin x(2+3\cos x)} dx$.

29. Solve the differential equation $ye^{\frac{x}{y}} dx = \left(xe^{\frac{x}{y}} + y^2 \right) dy$ ($y \neq 0$).

OR

Solve the differential equation: $(x^2 - 1) \frac{dy}{dx} + 2(x + 2)y = 2(x + 1)$.

30. By computing the shortest distance determine whether the pairs of lines intersect or not:

$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$

OR

If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that each one is perpendicular to the vector obtained by sum of the other two and $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = 5$, then prove that $|\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}$.

31. Maximize $Z = 100x + 170y$ subject to
 $3x + 2y \leq 3600$
 $x + 4y \leq 1800$
 $x \geq 0, y \geq 0$

SECTION – D (5 MARKS EACH)

32. Find the area of the region enclosed by the parabola $y^2 = x$ and the line $x + y = 2$.

33. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$, is an equivalence relation. Show that all the elements of $\{1, 3, 5\}$ are related to each other and all the elements of $\{2, 4\}$ are related to each other. But no element of $\{1, 3, 5\}$ is related to any element of $\{2, 4\}$.

OR

Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \Rightarrow B$ defined by $f(x) = \left(\frac{x-2}{x-3} \right)$. Is f one-one and onto? Justify your answer.

34. Given $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, find BA and use this to solve the system of equations $y + 2z = 7$, $x - y = 3$, $2x + 3y + 4z = 17$.

35. Find the shortest distance $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$ and $\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$.

OR

Find the image of the point with position vector $3\hat{i} + \hat{j} + 2\hat{k}$ in the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 4$. Also, find the position vectors of the foot of the perpendicular and the equation of the perpendicular line through $3\hat{i} + \hat{j} + 2\hat{k}$.

SECTION – E (4 MARKS EACH)

36. Read the following text carefully and answer the questions that follow:

A shopkeeper sells three types of flower seeds A_1, A_2, A_3 . They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.



Based on the above information:

- Calculate the probability that a randomly chosen seed will germinate. (1)
- Calculate the probability that the seed is of type A_2 , given that a randomly chosen seed germinates. (1)
- A die is thrown and a card is selected at random from a deck of 52 playing cards. Then find the probability of getting an even number on the die and a spade card. (2)

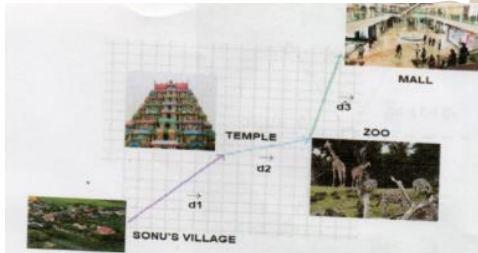
OR

If A and B are any two events such that $P(A) + P(B) - P(A \text{ and } B) = P(A)$, then find $P(A|B)$. (2)

37. Read the following text carefully and answer the questions that follow:

Sonu left from his village on weekend. First, he travelled d_1 displacement up to a temple. After this, he left for the zoo and travelled d_2 displacement. After this he left for shopping in a mall - Total driving time of Deepal from village to Mall was 1.5 hr.

If $d_1 = (6, 8)$, $d_2 = (3, 4)$ and $d_3 = (7, 12)$ km



- What is the total displacement from village to Mall? (1)
- What is the speed of Sonu from Village to Mall? (1)
- What is the Displacement from Village to Zoo? (2)

OR

What is the displacement from temple to Mall? (2)

38. Read the following text carefully and answer the questions that follow:

Mrs. Maya is the owner of a high-rise residential society having 50 apartments. When he set rent at ₹10000/month, all apartments are rented. If he increases rent by ₹250/month, one fewer apartment is rented. The maintenance cost for each occupied unit is ₹500/month.



- If P is the rent price per apartment and N is the number of rented apartments, then find the profit. (1)
- If x represents the number of apartments which are not rented, then express profit as a function of x. (1)
- Find the number of apartments which are not rented so that profit is maximum. (2)

OR

Verify that profit is maximum at critical value of x by second derivative test. (2)