

Class - 12TH

MATHEMATICS

MOCK EXAMS

Serial Number: 02

Time: 3 Hours

Maximum Marks: 80

Exam Date: _____



MATHEMATICS

MM. 80

12. For any two vectors $\vec{a}$ and $\vec{b}$ which of the following statements is always true?
 a. $\vec{a} \cdot \vec{b} < |\vec{a}||\vec{b}|$ b. $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$ c. $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$ d. $\vec{a} \cdot \vec{b} \geq |\vec{a}||\vec{b}|$
13. An ordered pair (α, β) for which the system of linear equations
 $(1 + \alpha)x + \beta y + z = 2$
 $\alpha x + (1 + \beta)y + z = 3$
 $\alpha x + \beta y + 2z = 2$
 has a unique solution is
 a. $(2, 4)$ b. $(-4, 2)$ c. $(-3, 1)$ d. $(1, -3)$
14. If for two events A and B, $P(A - B) = \frac{1}{5}$ and $P(A) = \frac{3}{5}$, then $P\left(\frac{B}{A}\right)$ is equal to
 a. $\frac{2}{5}$ b. $\frac{1}{2}$ c. $\frac{3}{5}$ d. $\frac{2}{3}$
15. Integrating factor of the differential equation $\frac{dy}{dx} + y \tan x - \sec x = 0$ is:
 a. $e^{\sec x}$ b. $e^{\cos x}$ c. $\sec x$ d. $\cos x$
16. If $|\vec{a}| = 4$ and $-3 \leq \lambda \leq 2$, then $|\lambda \vec{a}|$ lies in
 a. $[0, 12]$ b. $[8, 12]$ c. $[2, 3]$ d. $[-12, 8]$
17. For what value of k may the function $f(x) = \begin{cases} k(3x^2 - 5x), & x \leq 0 \\ \cos x, & x > 0 \end{cases}$ become continuous?
 a. 1 b. $-\frac{1}{2}$ c. 0 d. No value
18. Direction cosines of a line perpendicular to both x-axis and z-axis are:
 a. 0, 0, 1 b. 1, 1, 1 c. 1, 0, 1 d. 0, 1, 0
19. **Assertion (A):** If the circumference of the circle is changing at the rate of 10 cm/s, then the area of the circle changes at the rate 30 cm²/s, if radius is 3 cm.
Reason (R): If A and r are the area and radius of the circle, respectively, then rate of change of area of the circle is given by $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$.
 a. Both A and R are true and R is the correct explanation of A.
 b. Both A and R are true but R is not the correct explanation of A.
 c. A is true but R is false.
 d. A is false but R is true.
20. **Assertion (A):** If R is the relation defined in set $\{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$, then R is reflexive.
Reason (R): The relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is symmetric.
 a. Both A and R are true and R is the correct explanation of A.
 b. Both A and R are true but R is not the correct explanation of A.
 c. A is true but R is false.
 d. A is false but R is true.

SECTION – B (2 MARKS EACH)

21. $\sin^{-1}\left(\frac{-1}{2}\right)$

OR

Find the value of $\cos^{-1}\frac{1}{2} + 2 \sin^{-1}\frac{1}{2}$.

22. Show that the function given by $f(x) = \sin x$ is decreasing in $\left(\frac{\pi}{2}, \pi\right)$.

23. A particle moves along the curve $y = \left(\frac{2}{3}\right)x^3 + 1$. Find the points on the curve at which the y-coordinate is changing twice as fast as the x-coordinate.

OR

The total revenue in Rs received from the sale of x units of the product is given by $R(x) = 13x^2 + 26x + 15$. find Marginal Revenue when 17 unit are produce.

24. Evaluate: $\int e^x \cdot \left\{ \frac{2 + \sin 2x}{1 + \cos 2x} \right\} dx$.

25. If $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$ find $|A|$.

SECTION – C (3 MARKS EACH)

26. Using integration, find the area of the region bounded by the lines $x - y = 0$, $3x - y = 0$ and $x + y = 12$.
27. A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure.
28. Evaluate the definite integral $\int_0^\pi \frac{1}{1+\sin x} dx$.

OR

Evaluate: $\int \frac{1}{5-4\cos x} dx$.

29. Find the general solution of the differential equation: $(x^2 + 1) \frac{dy}{dx} - 2xy = (x^2 + 1)(x^2 + 2)$.

OR

Solve the differential equation: $[x\sqrt{x^2 + y^2} - y^2]dx + xydy = 0$.

30. Solve the Linear Programming Problem graphically:

Maximize $Z = x + y$ Subject to

$-2x + y \leq 1$

$x \leq 2$

$x + y \leq 3$

$x, y \geq 0$

OR

Determine graphically the minimum value of the objective function $Z = -50x + 20y$ subject to the constraints:

$2x - y \geq -5$

$3x + y \geq 3$

$2x - 3y \leq 12$

$x \geq 0, y \geq 0$

31. Show that the function $f(x)$ defined by $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x > 0 \\ 2, & x = 0 \\ \frac{4(1-\sqrt{1-x})}{x}, & x < 0 \end{cases}$ is continuous at $x = 0$.

SECTION – D (5 MARKS EACH)

32. Find the area of the region bounded by the curve $y = x^2$ and the line $y = 4$.
33. The scalar product of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ and hence, find the unit vector along $\vec{b} + \vec{c}$.

OR

Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{p}$, which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p} \cdot \vec{c} = 18$

34. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$

35. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\cos^{-1} \frac{1}{\sqrt{3}}$.

OR

A right circular cylinder is inscribed in a cone. Show that the curved surface area of the cylinder is maximum when the diameter of the cylinder is equal to the radius of the base of the cone.

SECTION – E (4 MARKS EACH)

36. Read the following text carefully and answer the questions that follow:
There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.



- i. What is the probability that the shell fired from exactly one of them hit the plane? (1)
- ii. If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B? (1)
- iii. What is the probability that the shell was fired from A? (2)

OR

How many hypotheses are possible before the trial, with the guns operating independently? Write the conditions of these hypotheses. (2)

37. Read the following text carefully and answer the questions that follow:

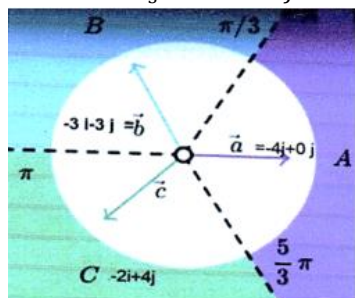
Three friends Ganesh, Dinesh and Ramesh went for playing a Tug of war game. Team A, B, and C belong to Ganesh, Dinesh and Ramesh respectively.

Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area (team areas shown below).

Team A pulls with $\vec{F}_1 = 4\hat{i} + 0\hat{j}$ KN

Team B $\rightarrow \vec{F}_2 = -2\hat{i} + 4\hat{j}$ KN

Team C $\rightarrow \vec{F}_3 = -3\hat{i} - 3\hat{j}$ KN



- i. Which team will win the game? (1)
- ii. What is the magnitude of the teams combine Force? (1)
- iii. What is the magnitude of the force of Team B? (2)

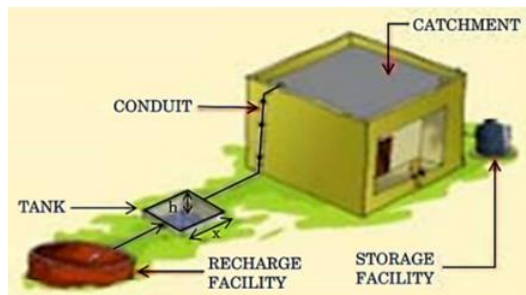
OR

How many KN Force is applied by Team A? (2)

38. Read the following text carefully and answer the questions that follow:

In order to set up a rain water harvesting system, a tank to collect rain water is to be dug. The tank should have a square base and a capacity of 250 m^3 . The cost of land is ₹ 5,000 per square metre and cost of digging increases with depth and for the whole tank, it is ₹ $40,000 h^2$, where h is the depth of the tank in metres. x is the side of the square base of the tank in metres.

ELEMENTS OF A TYPICAL RAIN WATER HARVESTING SYSTEM



- i. Find the total cost C of digging the tank in terms of x . (1)
- ii. Find $\frac{dC}{dx}$. (1)
- iii. Find the value of x for which cost C is minimum. (2)

OR

Check whether the cost function $C(x)$ expressed in terms of x is increasing or not, where $x > 0$. (2)